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Inflation with blowing-up solution of cosmological constant problem

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Abstract: The cosmological constant problem is how one chooses, without fine-tuning, one singular point $\Lambda_{\rm eff} = 0$ for the 4D cosmological constant. We argue that some recently discovered *weak self-tuning* solutions can be viewed as blowing-up this one point into a band of some parameter. These weak self-tuning solutions may have a virtue that only de Sitter space solutions are allowed outside this band, allowing an inflationary period. We adopt the hybrid inflation at the brane to exit from this inflationary phase and to enter into the standard Big Bang cosmology.

Keywords: Extra Large Dimensions, Cosmology of Theories beyond the SM, Physics of the Early Universe.

Contents

1. Introduction

There are three cosmological constants or vacuum energies which we try to understand now. The first and most difficult problem is understanding the vanishing cosmological constant at the minimum of the potential, which is the so-called cosmological constant problem, "Why is the cosmological constant zero at a natural scale of the Planck mass $M_{Pl} = 2.44 \times 10^{18}$ GeV?" [1, 2]. The second is the needed huge vacuum energy at the time of inflation [3]. The third is the present tiny vacuum energy $(0.003\,\mathrm{eV})^4$ observed by the supernova cosmology project [4]. Among these, the second one is understood by particle physics models of appropriate inflaton potentials. One example is the chaotic inflation that the vacuum energy at the time of inflation is huge during 60 or more e-folding time [5]. The third one is also understood by particle physics models under the name of quintessence [6].

At present, understanding the cosmological constant (c.c.) problem is the most difficult and unsolved problem. It is a fine-tuning problem in four-dimensional(4D) space-time. In 4D, in order to have the flat space the cosmological constant $\Lambda_{\rm eff}$ must be exactly zero, otherwise the space is curved. In higher dimensional space-time, this problem has another twist. The reason is that we require only the effective 4D space-time is flat after integrating out the extra space, even though the bulk is curved with a warp factor. Thus, with extra spaces there is more freedom. Indeed, the brane models *a la* Randall and Sundrum(RS) give some hope toward understanding this problem.

The recent self-tuning solutions of the cosmological constant problem are worked out in the RS type models in five dimensions(5D) [7]–[10]. These self-tuning solutions are distinguished by the *weak self-tuning solution* and the *strong self-tuning solution* [11]. The weak self-tuning solution requires just the existence of a 4D flat space solution consistent with the imposed boundary conditions. On the other hand, the strong self-tuning solution requires that the *4D flat space solution is the only solution* consistent with the boundary conditions. So far, there does not exist a strong self-tuning solution [7, 8]. But, there are examples of the weak self-tuning solutions: with antisymmetric tensor field H_{MNPQ} [9, 10, 11] in RS-II type models [12] and with the addition [13] of dilaton and the Gauss-Bonnet term [14].

The needed inflation in the brane world scenario is to obtain a de Sitter space solution at $t = -\infty$ and transform it into a flat space solution at $t = 0$ when the standard Big Bang cosmology commences. The study of time dependence is needed.

With the existing weak self-tuning solution, we studied the time dependence of a flat space at $t = -\infty$ to a flat space at $t = \infty$ as the brane tension Λ_1, located at $y = 0$, changes [10]. It will be possible to study the time dependence of a transition from a de Sitter space solution at $t = -\infty$ to a flat space solution at $t = \infty$, with weak self-tuning solution. At present we have not obtained a closed form de Sitter space solution with the weak self-tuning solution, and hence cannot study this kind of time dependence explicitly. But, it has been possible to study the time dependence of a flat space at $t = -\infty$ to a flat space at $t = \infty$ since we obtained closed form flat space solutions [10]. Therefore, it seems that it is not a trivial problem to introduce inflation with the self-tuning solutions.

In this paper, we consider inflation with the known closed form weak self-tuning solution [9]. In this Kim-Kyae-Lee(KKL) model, the flat solutions with a finite Planck mass is possible in a finite range of brane tension Λ_1,[1]

$$-\sqrt{-6\Lambda_b} < \Lambda_1 < \sqrt{-6\Lambda_b} \tag{1.1}$$

where Λ_b is the AdS bulk c.c., and we set the 5D fundamental mass $M = 1$. Therefore, one anticipates that the flat solutions are not possible at $|\Lambda_1| > \sqrt{-6\Lambda_b}$ where however de Sitter space solutions may be possible. If so, a sufficient inflation is possible in the de Sitter space. Then, by letting the brane tension Λ_1 fall in the range of eq. (1.1), one can terminate the inflation and reaches a flat space. One such example is the hybrid inflation [15] at the brane.

This idea is sketched in figure 1. The cosmological constant problem in 4D is a fine-tuning problem that the flat space is possible only for $\Lambda_{\text{eff}} = 0$ as shown in figure 1(a). Only at this one point the flat space is obtained. In our self-tuning solutions [9, 10, 11], the flat spaces are possible in a finite range of 5D parameters, as shown in (1.1). This can be viewed as a blowing-up of the one point, as shown in figure 1(b). One notable feature is that the KKL model does not allow flat space solutions outside the range given in (1.1), namely the inflation can be naturally introduced in the range of parameters, $|\Lambda_1| > \sqrt{-6\Lambda}$. Exit from inflation is achieved by a hybrid inflation at the brane B1, by making Λ_1 fall

[1] This idea can be used in any self-tuning model with a band of blowing-up solutions.

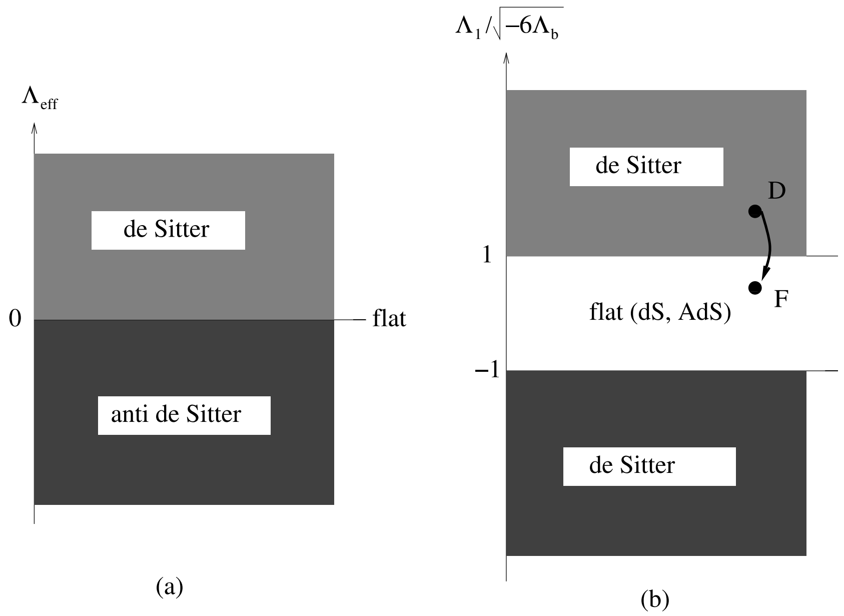

Figure 1: The blowing-up of the 4D singular point $\Lambda_{\rm eff} = 0$ (a) into a finite range of 5D parameter Λ_1 (b).

in the region allowed in eq. (1.1). In this case, it is not a fine-tuning since it is possible to have a flat space in a finite range of Λ_1. The KKL solution, however, is a weak self-tuning solution since it allows dS and AdS solutions also within the band (1.1). If a strong self-tuning solution were found with the above type of a blowing-up mode, it will be much more satisfactory.[2]

In section 2, we review some salient features of the KKL solution, which is needed for later discussions. In section 3 we show that there exists the de Sitter space solutions outside the region of eq. (1.1). In section 4, we discuss the hybrid inflation at the brane so that Λ_1 falls inside the region given in (1.1) after a sufficient inflation. In section 5, realization of zero cosmological constant is discussed. Section 6 is a brief conclusion.

2. The weak self-tuning solution: blowing-up model of the cosmological constant singularity

There are weak self-tuning solutions in 5D [9, 11, 13]. Among these we choose the KKL model [9] where a closed form self-tuning solution was obtained. In this model, we introduce a three-form field A_{MNP} $(M, N, P = 0, 1, \ldots, 4)$, where its field strength is H_{MNPQ}. The relevant action is[3]

$$S = \int d^4x \int dy \sqrt{-g} \left(\frac{1}{2} R + \frac{2 \cdot 4!}{H^2} - \Lambda_b + \mathcal{L}_m \delta(y) \right) \tag{2.1}$$

[2]Even if the strong self-tuning solution of ref. [7] works without the problem pointed out in ref. [8], it does not belong to the class of blowing-up solutions.

[3]In ref. [11], another closed form is given with a logarithmic function of H^2.

where y is the fifth coordinate, $H^2 = H_{MNPQ}H^{MNPQ}$, and we set the fundamental mass parameter $M = 1$ which can be recovered when its explicit expression is needed. Also, we assume a Z_2 symmetry of the solution, $\beta(-y) = \beta(y)$. The sign of $1/H^2$ was chosen such that at the vacuum the propagating field A_{MNP} has a standard kinetic energy term.

2.1 Flat space solutions

The flat space ansatz for the metric is

$$ds^2 = \beta^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{2.2}$$

where $(\eta_{\mu\nu}) = \mathrm{diag}(-1, +1, +1, +1)$. The Einstein tensors are

$$G_{\mu\nu} = g_{\mu\nu}\left[3\left(\frac{\beta'}{\beta}\right)^2 + 3\left(\frac{\beta''}{\beta}\right)\right], \qquad G_{55} = 6\left(\frac{\beta'}{\beta}\right)^2 \tag{2.3}$$

where prime denotes differentiation with respect to y. The energy-momentum tensors are

$$T_{MN} = -g_{MN}\Lambda_b - g_{\mu\nu}\delta^\mu_M\delta^\nu_N\Lambda_1\delta(y) + 4\cdot 4!\left(\frac{4}{H^4}H_{MNPQ}H_N{}^{PQR} + \frac{1}{2}g_{MN}\frac{1}{H^2}\right). \tag{2.4}$$

The field equation of H,

$$\partial_M\left(\sqrt{-g}\frac{H^{MNPQ}}{H^4}\right) = 0\,, \tag{2.5}$$

is satisfied with the following y dependence of the four-form field

$$H_{\mu\nu\rho\sigma} = \sqrt{-g}\frac{\epsilon_{\mu\nu\rho\sigma}}{n(y)}\,. \tag{2.6}$$

The brane B1 located at $y = 0$ has a δ-function discontinuity of $\beta'(0)$, which dictates the following boundary condition at $y = 0^+$,

$$\left.\frac{\beta'}{\beta}\right|_{y=0^+} = -\frac{\Lambda_1}{6}\,. \tag{2.7}$$

Then, the flat solution is given by

$$\beta(|y|) = \left(\frac{k}{a}\right)^{\frac{1}{4}}\frac{1}{\cosh(4k|y| + c)^{1/4}} \tag{2.8}$$

where

$$k = \sqrt{-\frac{\Lambda_b}{6}}\,, \qquad a = \sqrt{\frac{1}{6A}} \tag{2.9}$$

with a positive constant $A > 0$, and c is an integration constant to be determined by the boundary condition (2.7).

The condition (2.7) determines c as

$$c = \tanh^{-1}\left(\frac{\Lambda_1}{\sqrt{-6\Lambda_b}}\right) \tag{2.10}$$

which allows the flat solution only in the interval

$$-\sqrt{-6\Lambda_b} < \Lambda_1 < \sqrt{-6\Lambda_b}\,. \tag{2.11}$$

One important point of the KKL solution summarized above is that the *flat 4D space is possible only in the band of* Λ_1 shown in eq. (2.11). One can envision this situation in 5D as a blowing-up of a singular point $\Lambda_{\rm eff} = 0$ of the *flat* 4D case, as sketched in figure 1.

The 4D Planck mass is expressed as

$$M^2_{\rm P,eff} = 2M^3\sqrt{\frac{k}{a}}\int_0^\infty dy \frac{1}{\sqrt{\cosh(4ky+c)}}\,. \tag{2.12}$$

Thus, the order of the 4D Planck mass squared is $M^3/\sqrt{ka}$.

2.2 De Sitter and anti de Sitter space solutions

The action (2.1) allows the de Sitter and anti de Sitter space solutions. The ansatz for these curved spaces are taken as

$$ds^2 = \beta^2(y)\bar{g}_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{2.13}$$

where

$$\bar{g}_{\mu\nu} = {\rm diag}\left(-1, e^{2\sqrt{\bar\Lambda}t}, e^{2\sqrt{\bar\Lambda}t}, e^{2\sqrt{\bar\Lambda}t}\right), \qquad (dS_4 \text{ background}, \bar\Lambda > 0) \tag{2.14}$$

and

$$\bar{g}_{\mu\nu} = {\rm diag}\left(-e^{2\sqrt{-\bar\Lambda}t}, e^{2\sqrt{-\bar\Lambda}t}, e^{2\sqrt{-\bar\Lambda}t}, 1\right), \qquad (AdS_4 \text{ background}, \bar\Lambda < 0)\,. \tag{2.15}$$

One important equation, relating $\beta(y)$ and $\beta'(y)$, is

$$\left|\frac{\beta'}{\beta}\right| = \sqrt{\frac{\bar{k}^2}{\beta^2} + k^2 - a^2\beta^8} \tag{2.16}$$

where the 4D effective curvature $\bar{k}$ and the bulk curvature k are

$$\bar{k} = \sqrt{\frac{\bar\Lambda}{6}}, \qquad k = \sqrt{\frac{-\Lambda_b}{6}} \tag{2.17}$$

It was shown that there exist de Sitter and anti de Sitter space solutions [10]. It can be studied by investigating eq. (2.16) in the limit of $\beta \to 0$ or $\beta' \to 0$ [11].

2.3 Rescaling of $\beta(0)$

We can rescale $\beta(0)$ for our convenience. But this rescaling is possible due to our implicit *assumption that the effective 4D Planck mass is the definition of a scale.* If a trans-Plankian physics is present, then the defining scale can be given at a higher-than-Planck scale and the following discussion on the rescaling of $\beta(0)$ is not allowed.

Let us see how the parameters are changed under the rescaling, in the de Sitter space for an explicit presentation. Defining $\tilde{\beta}$ as

$$\tilde{\beta}^2 = \frac{\beta^2(y)}{\beta^2(0)}, \qquad \tilde{\beta}^2(0) = 1, \tag{2.18}$$

we can consistently redefine the space-time coordinates,

$$\tilde{t} = |\beta(0)| t, \qquad \tilde{x}^i = |\beta(0)| x^i, \tag{2.19}$$

such that the metric (2.14) takes the following form,

$$ds^2 = \tilde{\beta}^2(y)\left(-d\tilde{t}^2 + d\tilde{x}^i d\tilde{x}^i e^{2\sqrt{\bar{\Lambda}}\frac{\tilde{t}}{|\beta(0)|}}\right) + dy^2. \tag{2.20}$$

The curvature in the new coordinate system is

$$\tilde{\Lambda} = \frac{\bar{\Lambda}}{\beta^2(0)}. \tag{2.21}$$

The 4D Planck mass in the tilded coordinate is given by

$$\tilde{M}^2_{\rm P,eff} = \frac{M^2_{\rm P,eff}}{\beta^2(0)} \tag{2.22}$$

where

$$M^2_{\rm P,eff} = 2M^3 \int_0^\infty dy \beta^2(y). \tag{2.23}$$

Therefore, the curvature in units of the Planck mass is the same in the untilded and tilded coordinates,

$$\frac{\bar{\Lambda}/M^2}{M^2_{\rm P,eff}} = \frac{\tilde{\Lambda}/M^2}{\tilde{M}^2_{\rm P,eff}}. \tag{2.24}$$

Now, the derivative equation (2.16) becomes,

$$\left|\frac{\tilde{\beta}'}{\tilde{\beta}}\right| = \sqrt{\frac{\tilde{k}^2}{\tilde{\beta}^2} + k^2 - a^2\beta^8(0)\tilde{\beta}^8} \tag{2.25}$$

where

$$\tilde{k}^2 = \frac{\bar{k}^2}{\beta^2(0)}. \tag{2.26}$$

Thus, the derivative at 0^+ is given by

$$|\tilde{\beta}'|_{0^+} = \sqrt{\tilde{k}^2 + k^2 - a^2\beta^8(0)}. \tag{2.27}$$

3. Existence of de-Sitter-space-only region

In this section, we study time dependent but spatially homogenious 4D solutions which can be useful for cosmological studies.

3.1 The nearby flat and curved space solutions

To compare the nearby solutions, it is convenient to use the tilded coordinate where $\tilde{\beta}(0)$ is defined to be 1. Let us consider the nearby solutions for the flat, de Sitter(dS), and anti de Sitter(AdS) spaces. In this case, three solutions will be almost identical up to the point $y = (\textit{very large})$ so that the flat solution tail is negligible. The dS space solution has the horizon(the point where $\tilde{\beta}(y_h) = 0$) at $y_h \simeq \infty$. Also, the AdS space solution has the first cycle(up to the point where $\tilde{\beta}'(y_A) = 0$ and $\tilde{\beta}(y_A) = \epsilon^+$) extending almost to $y_A \simeq \infty$. The three cases of these nearby solutions are depicted in figure 2. Since the AdS solution can be discussed similarly as for the dS case, we compare below the flat and dS solutions only.

In the tilded coordinate, it is easy to compare the nearby solutions since $\tilde{\beta}(0)$ is defined to be 1. The boundary condition at B1, (2.27), should be the same for the flat and dS solutions since we look for solutions with the same Λ_1. Thus, if both flat and dS solutions exist, we have an equality,

$$\sqrt{k^2 - a_{\rm F}^2\beta_{\rm F}^8(0)} = \sqrt{\epsilon + k^2 - a_{\rm dS}^2\beta_{\rm dS}^8(0)} \tag{3.1}$$

where ϵ is the infinitesimal curvature $\delta\tilde{k}^2$ near the flat solution, and subscripts F and dS denote the respective values in the flat and dS space solutions. We can choose the integration constant $a_{\rm dS}^2$ such that $a_{\rm dS}^2\beta_{\rm dS}^8(0) = a_{\rm F}^2\beta_{\rm F}^8(0) + \epsilon$, satisfying the boundary condition (3.1). Thus, we conclude that there exists a nearby dS space solution close to the flat space solution, which means that in the band for the flat solutions (2.11) there exist de Sitter space solutions. A similar conclusion can be drawn for AdS solutions. Therefore, in the region where flat space solutions are allowed, the dS and AdS solutions are also allowed. It was shown previously that these dS and AdS solutions exist by studying the behavior of β' in ref. [10].

3.2 De Sitter space solutions at $|\Lambda_1| > \sqrt{-6\Lambda_b}$

Let us rewrite (2.25) as,

$$|\tilde{\beta}'| = \sqrt{\tilde{k}^2 + k^2\tilde{\beta}^2 - \tilde{a}^2\tilde{\beta}^{10}} \tag{3.2}$$

where

$$\tilde{a}^2 = a^2\beta^8(0)\,. \tag{3.3}$$

We know that dS space solutions are possible if

$$\tilde{\beta}(y_h) = 0\,, \qquad \tilde{\beta}'(y_h) \neq 0 \tag{3.4}$$

are satisfied [10, 11]. Note that the discontinuity condition of the derivative of β at $y = 0$ is

$$\left|\frac{\tilde{\beta}'}{\tilde{\beta}}\right|_{0^+} = \sqrt{\tilde{k}^2 + k^2 - \tilde{a}^2} = \left|\frac{\Lambda_1}{6}\right|.$$

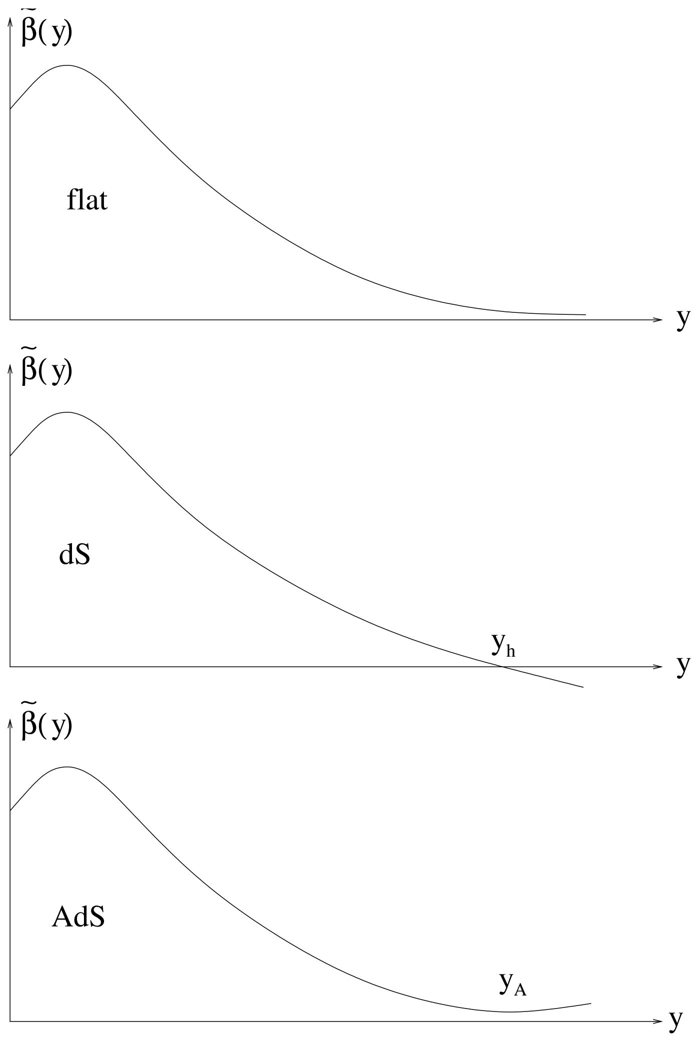

Figure 2: The schematic behaviors of nearby flat, de Sitter, and anti de Sitter space solutions. The numerical values of M_{Pl} are almost the same.

Thus, in the region where flat space solutions are forbidden, we obtain an inequality from $\Lambda_1/6 > \sqrt{-\Lambda_b/6}$,

$$\sqrt{\tilde{k}^2 + k^2 - \tilde{a}^2} > \sqrt{\frac{-\Lambda_b}{6}}$$

which is

$$\tilde{k}^2 > \tilde{a}^2\,, \tag{3.5}$$

or the effective 4D curvature $\bar{\Lambda}$ is bounded as

$$\bar{\Lambda} > 6a^2 M^2 \beta^{10}(0) \tag{3.6}$$

where we recovered M^2. Thus, we conclude that in the region $|\Lambda_1| > \sqrt{-6\Lambda_b}$ both flat and AdS solutions are forbidden. *Only dS solutions are allowed with the curvature bounded from below.* Note that the de Sitter space solution is obtained for the negative tension also,

$\Lambda_1 < -\sqrt{-6\Lambda_b}$. In this case, the 4D gravity at the brane is antigravity and one must be careful in studying cosmological effects at the negative tension brane [16, 17].

To estimate the order of the magnitude, let us observe that the flat space solution (2.8) shows that $a^2\beta^8(0)$ is proportional to k^2. Thus, we guess that a rough inequality is $\bar{\Lambda} > 6M^2k^2\beta^2(0)$, or

$$\bar{\Lambda} > -\frac{\Lambda_b}{M}\beta^2(0)\,. \tag{3.7}$$

4. Hybrid inflation at the brane

Consider the de Sitter space solution where $\Lambda_1 > \sqrt{-6\Lambda_b}$, i.e. the positive tension brane outside the region given in eq. (2.11). In the previous section, in this region of Λ_1 it was shown that the dS space solution is the only possibility. Also, the conventional 4D Einstein equations are obtained at the positive tension brane [16]. This positive tension 3-brane is called B1. Now we can treat Λ_1 as the vacuum energy at B1. Namely, Λ_1 here is the vacuum energy of the lagrangian given in eq. (2.1). Therefore, the field evolution at B1 can be studied as in the usual 4D cosmology.

Any reasonable inflation model can be adopted to transform the dS-only region to the region given in (2.11). Probably, the most modern and attractive inflationary model is the *hybrid inflation* [15, 18] which we adopt here for the purpose of lucid presentation. In the hybrid inflation, at least two fields participate in the inflationary period. One field ϕ(the inflaton field) is responsible for keeping the potential flat during enough e-foldings and another field ψ(the waterfall field) is responsible for exiting the inflationary period by radiating light particles. Below we discuss cosmology at brane B1.

Since it is sufficient to present our idea with the simplest original model of hybrid inflation [15, 19], we adopt this model in this paper. The coupled potential of the inflaton and waterfall fields is assumed as

$$\begin{aligned} V &= \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda(\psi^2-\mu^2)^2 + \frac{1}{2}\lambda'\psi^2\phi^2 \\ &= V_0 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}(-m_\psi^2+\lambda'\phi^2)\psi^2 + \frac{1}{2}\lambda'\psi^2\phi^2 \end{aligned} \tag{4.1}$$

where $V_0 = \frac{1}{4}m_\psi^2\mu^2, m_\psi^2 = \lambda\mu^2$, and we treat μ^2 and m^2 as large and small parameters, respectively. Studying the effective mass of the waterfall field ψ, we can see immediately that ψ sits at $\psi = 0$ if the inflaton field ϕ is greater than the critical value ϕ_c,[4]

$$\phi_c \equiv \sqrt{\frac{\lambda}{\lambda'}}\,\mu\,. \tag{4.2}$$

If ϕ moves below the critical value, the waterfall field ψ feels the tachionic potential and rolls down to the minimum at $\psi = \mu$. It was shown that the time needed to roll down the hill after ϕ passes ϕ_c is of order one Hubble time [19]. In the region $|\phi| \gg \phi_c$, the inflaton potential looks like $V_0 + \frac{1}{2}m^2\phi^2$, and for a small value of m^2 a sufficient inflation is possible. One can take $m \sim$ TeV.

[4]We will consider the positive ϕ region during the inflationary period.

During the inflationary period, the height of the potential is of order

$$V_0 = \frac{\lambda}{4}\mu^4 \,. \tag{4.3}$$

During the inflation, we require an almost flat potential, implying $m^2 \ll \frac{1}{2}\lambda'\mu^2$. Namely, we require that μ^2 is a large parameter and m^2 is a small parameter. At the time when ϕ reaches ϕ_c, the Hubble parameter becomes

$$H^2 = \frac{\lambda\mu^4}{12M_{Pl}^2} \,. \tag{4.4}$$

Requiring m smaller than the Hubble parameter, $m^2 \ll H^2$, we have

$$\mu^2 \gg \sqrt{\frac{12}{\lambda}}\, mM_{Pl} \,. \tag{4.5}$$

With this condition a sufficient inflation is guaranteed. Furthermore, it was argued that even a value of $m^2 \sim O(H^2)$ does not ruin the needed inflation [19, 20].

The condition of forbidding the flat space solution, namely the condition for inflation, is

$$|\Lambda_1| > \sqrt{6M^3|\Lambda_b|} \,, \tag{4.6}$$

which is equivalent to

$$\mu^4 > \sqrt{\frac{96}{\xi\lambda^2}M_{Pl}^2 k_a|\Lambda_b|} \tag{4.7}$$

where

$$k_a \equiv \sqrt{ak}, \tag{4.8}$$

and ξ is the integral in the definition of $M_{\rm P,eff}^2 = M_{Pl}^2$ in eqs. (2.12) and (2.23)

$$M_{Pl}^2 = \xi\frac{M^3}{k_a} \,. \tag{4.9}$$

$\xi_{\rm flat}(c)$ of the flat space solution is estimated as

$$\xi_{\rm flat}(c) = \frac{1}{2}\int_0^\infty dx \frac{1}{\sqrt{\cosh(x+c)}} \to \xi_{\rm flat}(0) = \sqrt{\frac{8}{\pi}}\left(\Gamma\left(\frac{5}{4}\right)\right)^2 \simeq 1.311 \,. \tag{4.10}$$

If we consider the parameters in the range, $\xi \simeq 1$, $\lambda \simeq 1$, $M = [10^{16}\,{\rm GeV},\, 10^{18}\,{\rm GeV}]$, then k_a falls in the range $[1.7\times 10^{11}\,{\rm GeV},\, 1.7\times 10^{17}\,{\rm GeV}]$. Thus, the inflation condition becomes

$$\mu > [1.3\times 10^{13}\,{\rm GeV}, 1.8\times 10^{18}\,{\rm GeV}] \,. \tag{4.11}$$

As an eyeball number, let us take

$$-\Lambda_b = 10^{14}\,{\rm GeV}\,, \qquad k_a = 10^{14}\,{\rm GeV}\,, \qquad \xi = 1\,, \qquad \lambda = 1\,. \tag{4.12}$$

Then

$$M = 8.4\times 10^{16}\,{\rm GeV}\,, \qquad \mu > 2.2\times 10^{15}\,{\rm GeV}\,. \tag{4.13}$$

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From eq. (3.7), we can estimate the allowed vacuum energy V_0 which must be bounded,

$$V_0 \sim \bar{\Lambda} > (1.86 \times 10^{13}\,\mathrm{GeV})^4 \beta^2(0). \tag{4.14}$$

Since we take $\mu > 2.2 \times 10^{15}$ GeV (note that $V_0 = \frac{\lambda}{4}\mu^4$), it is consistent with the above condition for the de Sitter space curvature $\bar{\Lambda}$ in the de-Sitter-space-only region. During the inflation, the vacuum energy is huge and there does not arise the possible problems encountered in TeV inflationary models [21].

Thus, there is a reasonable range of parameters satisfying the inflationary condition.

After a sufficient inflation in the effective 4D cosmology, the brane tension at B1 changes when the brane field inflaton ϕ becomes smaller than the critical value ϕ_c. Within an order of the Hubble time at $\phi \sim \phi_c$, the brane vacuum energy or the brane tension Λ_1 falls in the range of eq. (2.11) which allows flat space solution. Within this range, however, not only the flat solutions but also curved solutions are possible. If the dS space solution is chosen, a further inflation would result. On the other hand, if a flat space solution is chosen, we exit from inflation and go into the standard Big Bang cosmology. The most probable case is that a flat space is chosen and inflation ends as in the standard hybrid inflation scenario in the flat space background. The question is how the flat space solution is chosen after Λ_1 falls in the range given in eq. (2.11).

Before closing this section, we discuss the issues raised in brane cosmologies [22].

To compare with some formulae of ref. [22], we work in the dual picture [23] where a pseudoscalar σ is the dual of $H_{MNPQ}, \partial_M\sigma \propto \epsilon_{MNPQR}H^{NPQR}$. Then, the exact Hubble equation at the brane is given by

$$\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{\Lambda_b}{3} - \frac{2\gamma}{3b_0^2}\left[\frac{1}{3}((\partial\sigma_0)^2)^{-\frac{2}{3}}\sigma_0'^2 - \frac{1}{2}b_0^2((\partial\sigma_0)^2)^{\frac{1}{3}}\right] - \frac{1}{36}\rho(\rho+3p) \tag{4.15}$$

where $\gamma = 6(4!)^{4/3}$, the 5D fundamental scale is set at $M = 1$, the subscript 0 means the value at the brane located at $y = 0$, and ρ and p are the total energy density and pressure, respectively, on the brane. Expressing the brane energy density and pressure in terms of the brane tension Λ_1 and the rest as $\rho = \Lambda_1 + \rho_{dyn}$ and $p = -\Lambda_1 + p_{dyn}$, and assuming that $\sigma_0'(t,0) = \bar{\sigma}_0'(0)$ and $b_0 = 1$ as in the flat solution, the Hubble equation at the brane (4.15) with the self-tuning solution [9] becomes [24]

$$\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2 = 2k^2\left[(\mathrm{sech}(c))^2 - 1 + \left(\frac{k_1}{k}\right)^2\right] + \frac{1}{36}[\Lambda_1(\rho_{dyn} - 3p_{dyn}) - \rho_{dyn}(\rho_{dyn} + 3p_{dyn})] \tag{4.16}$$

where c is the integration constant to be determined from the boundary condition at the brane to give a flat solution. In eq. (4.16), the first part on the right hand side vanishes for an arbitrary set of (k, k_1) for the self-tuning solution. The second part on the right hand side contains two pieces, one with the linear term(i.e. the one with Λ_1) and the other with quadratic terms of ρ_{dyn} and p_{dyn}. The linear term in our case is different from that of Kachru et al's model [22], where the linear term is proportional to $\rho_{dyn} + p_{dyn}$. For an application to Big Bang cosmology, the second term on the right hand side has to be scrutinized. In the matter dominated era, we obtain the needed linear term as a dominant one. In the usual radiation dominated universe, we have the standard relation $p_{dyn} = \frac{1}{3}\rho_{dyn}$. With this relation, the right hand side of (4.16) gives a small negative term which does not solve eq. (4.16). This implies that when the energy and pressure becomes small compared to Λ_1 the thermal equilibrium condition would give $p_{dyn} = \frac{1}{3}\rho_{dyn} - \epsilon$ with a small number $\epsilon > 0$, not exactly reproducing the standard equilibrium relation between energy and pressure, $p_{dyn} = \frac{1}{3}\rho_{dyn}$, of the radiation dominated universe. The departure from the standard relation is coming from the effect of gravity sector at the brane. But this departure from the usual relation is tiny in our model, i.e. at the time of nucleosynthesis($T = 1\,\mathrm{MeV}$) ϵ is of order $\sim \rho_{dyn}/\Lambda_1 < 10^{-64}$. Thus, we obtain a radiation dominated universe with the discrepancy from the standard energy and pressure relation astronomically small.

5. Vanishing cosmological constant

In figure 1, we sketched the parameter change from Point D to Point F. Point D allows only the de Sitter space and inflation is achieved without any difficulty. However, in the KKL model Point F allows flat, de Sitter and anti de Sitter space solutions. Only when a flat space is chosen, we obtain the vanishing cosmological constant. In this regard, obtaining a strong self-tuning solution with a blowing-up mode is of utmost importance. With a strong self-tuning solution, then Point F allows only flat space solutions and we arrive at the zero cosmological constant vacuum after exiting from the inflationary phase.

With the weak self-tuning solution as discussed in this paper, we must rely on an additional principle to choose flat space solutions. In ref. [10], Hawking's probabilistic selection [25] was used. With the present form of hybrid inflation, this probabilistic choice can be more elegantly phrased. The initial condition for the universe is the state D in figure 1(b). With this initial condition, it is unavoidable to have an inflationary period. When the inflaton field ϕ passes the point ϕ_c, the parameter set is given such that the band of eq. (1.1) is chosen. Now there are infinite ways to choose classical pathes, starting from Point D and ending at Point F. The probability to end at F with a flat solution is infinitely larger than the probability to end with dS or AdS solutions. It is most probable that flat space solutions are chosen as soon as the path enters in the band of eq. (1.1). Hawking's argument [25] can be repeated in this situation. The euclidean space integral involves $\int dy \int d^4x_E(\cdots)$ where $(\cdots)$ represent the relevant vacuum energy. This integral is maximum where a flat space is chosen at every point of y, which means that as soon as the bound (1.1) is satisfied a flat space is chosen and continues to be so. The flat space obtained in this way corresponds to the field values where equations of motion are satisfied, presumably at the minimum of the potential. The above argument is notably qualitative. One can question, "What is the dominant sourse for ending the inflation?" We can say qualitatively that it is the waterfall field in the flat background. It will be interesting to see more accurately how the inflation terminates.

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Thus, the present tiny vacuum energy above the point of a true minimum in the flat background will show up after the matter energy becomes comparable to dark energy $(0.003\,\mathrm{eV})^4$. Eventually, the universe feels the hill of this tiny tiny potential and will give the vanishing cosmological constant as soon as this tiny hill provides an oscillation[5] and this oscillating mode behaves like cold dark matter.

6. Conclusion

We introduced a reasonable scenario for a *sufficient inflation* and transition to a vacuum with the *vanishing cosmological constant* in RS-II type models. Here, the observable sector fields are located at the brane B1. The necessary ingredient is the de-Sitter-space-only region in a blowing-up solution of the cosmological constant problem. We discussed the scheme with a previously found blowing-up solution in the form of weak self-tuning solution [9]. This idea can be applied to any blowing-up solution of the cosmological constant problem with a de-Sitter-space-only region. Inflation is unavoidable for some initial conditions of the universe due to the existence of de-Sitter-space-only region. Fine-tuning is avoided because the flat space solutions are allowed in a finite range of parameter Λ_1. Exit from inflation can be realized in any reasonable inflationary model. In particular, we showed that it is possible to introduce a hybrid inflation, being consistent with the blowing-up region of the model we discussed. Since the example we choose for the blowing-up solution is a weak self-tuning solution, we must rely on another principle to choose the flat space after inflation. For this we adopted Hawking's probabilistic argument.

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